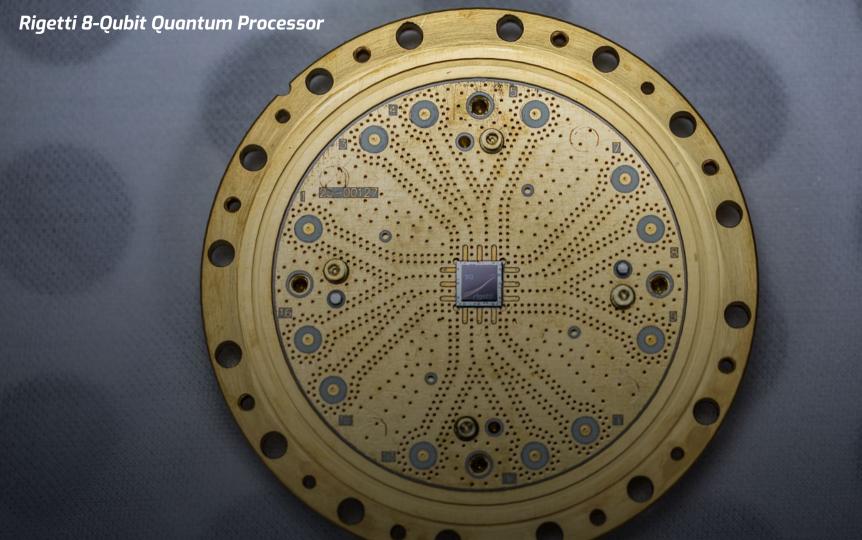


Quantum Cloud Computing
Johannes Otterbach

TU Kaiserslautern January 22, 2018







- > Scalable Gate-model Quantum Processors
 - > Superconducting Microwave Circuits
- > Focus on near-term applications
 - > Quantum/Classical Hybrid Algorithms
- > Build towards fault-tolerance
- > Access over the cloud
 - > Quantum computers as co-processors

Rigetti 8-Qubit Quantum Processor Founded in 2013 by **Chad Rigetti** Fab-1: Fremont, CA ~100 Employees Forest: Quantum computing over the cloud

Venture back

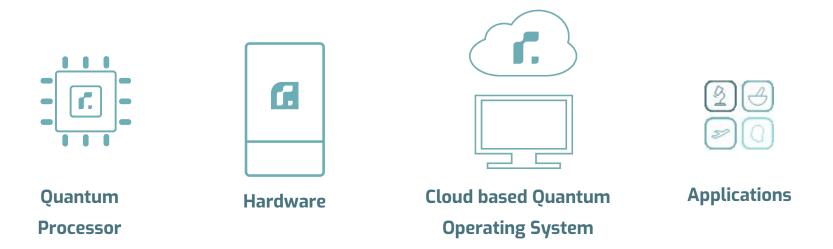
R&D lab: Berkeley, (

Full-stack scalab superconduction qubi

30-qubit Quantu Virtual Machine



Full Stack Quantum Computing





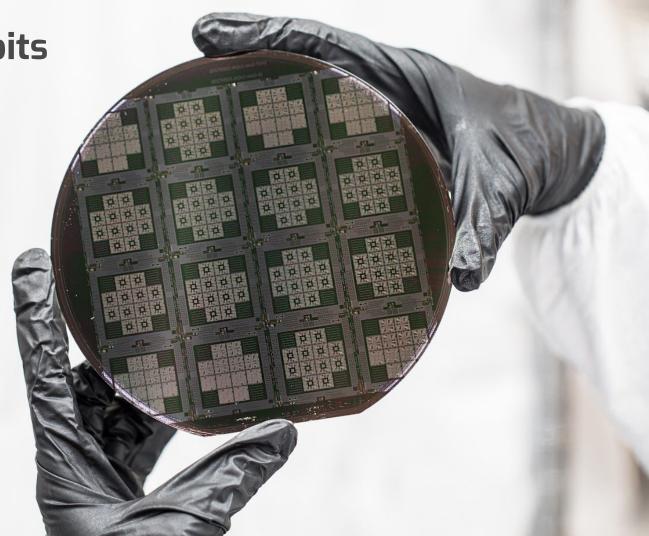
What do our **Qubits** look like?

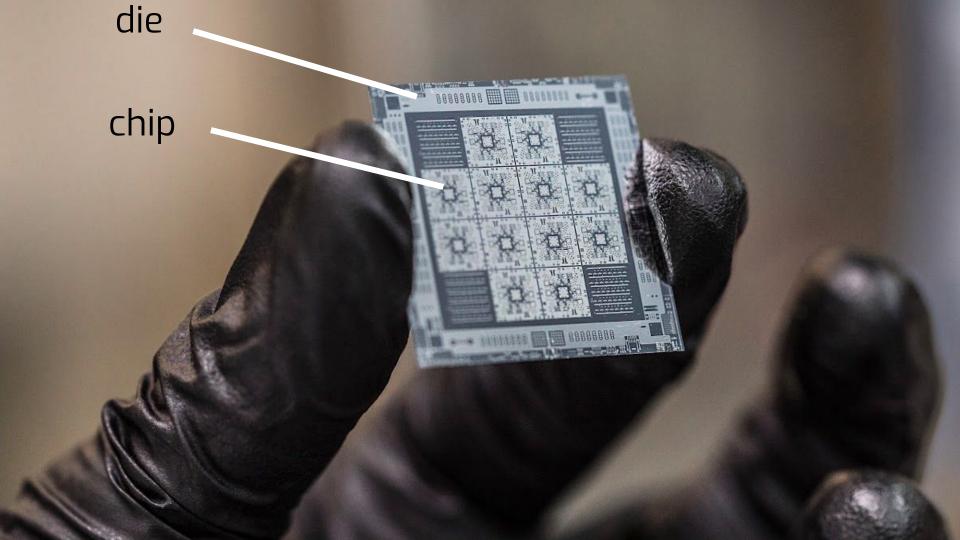
Superconducting circuits

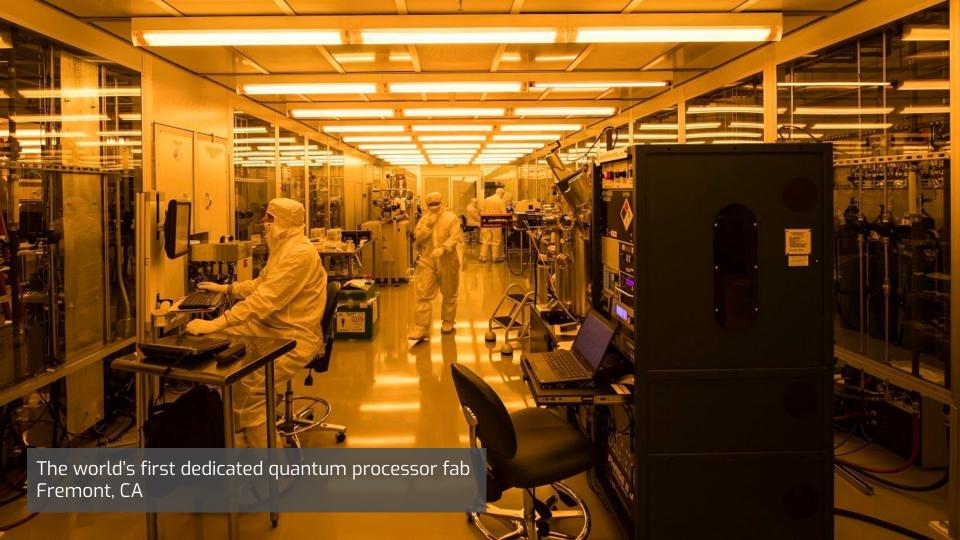
Operated near absolute zero

Aluminum on silicon

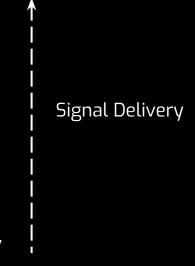
Microwave signal delivery



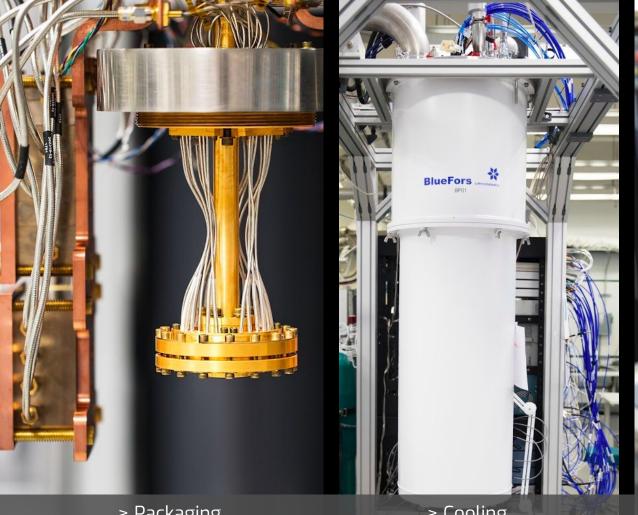








The Chip





> Packaging

> Cooling

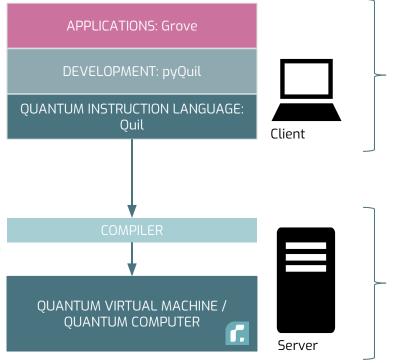
> Control Electronics

FOREST: Tools for experimental quantum programming

forest.rigetti.com

- > Write applications...
- > using tools...
- > that build quantum programs...

- > that compile onto quantum hardware...
- that execute on a real or virtual quantum processor.



Open-sourced on github under Apache v2.0 license

github.com/rigetticomputing/pyquil

github.com/rigetticomputing/grove

Simulator in public-beta Quantum HW

forest.rigetti.com



Quantum Algorithms

Simulating quantum systems

1981

Original proposal by Feynman: Simulating physics with computers

"The physical world is quantum mechanical, and therefore the proper problem is the simulation of quantum physics."

Quantum Algorithms

Shor's factoring algorithm (Shor) 1995 Phase Estimation (PE) introduced (Kitaev) Hamiltonian Simulation by PE (Lloyd) ----- 2002 Map of fermions to paulis (Somma) ----- 2005 Molecular ground states w/ PE (Aspuru-Guzik) 2010 H₂ ground state using simulated QC (Whitfield) ---- 2013 Hybrid Quantum-Classical Algorithms VQE (McClean) 2015 Approximate Combinatorial Optimization (Farhi)

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2013

2013

Molecular ground states w/ VQE Implementation on a photonics processor (Peruzzo)

2014

Quantum Combinatorial Optimization (QAOA)

(Farhi), eg. MAX-CUT, MaxE3Lin2

2015

VQE + PE on superconducting qubits, Small quantum machine learning examples

----**2016**

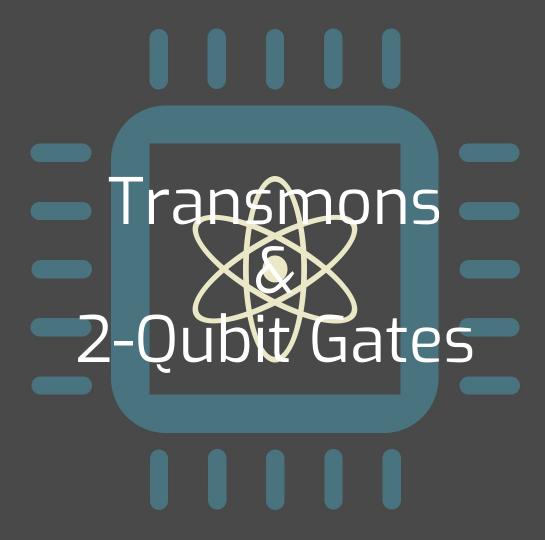
Broader applications of VQE (Troyer, Rubin)

----**• 2017**

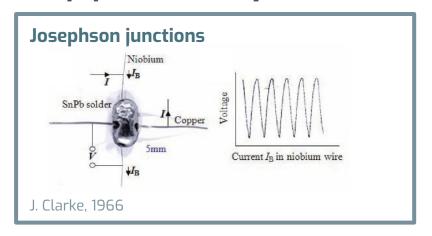
Machine Learning (Aspuru-Guzik, Otterbach)

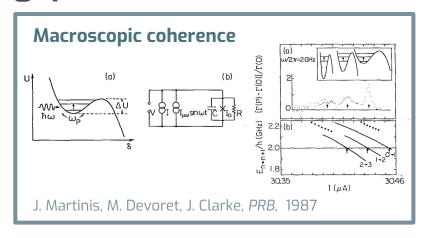
TODAY

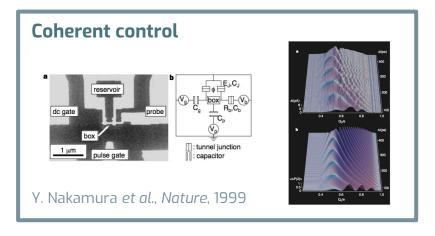
Simulating quantum systems

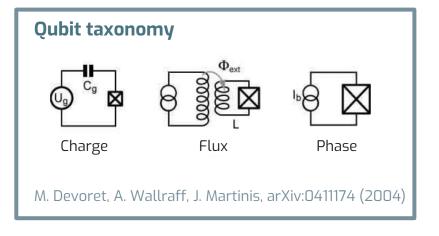


Early years of superconducting qubits





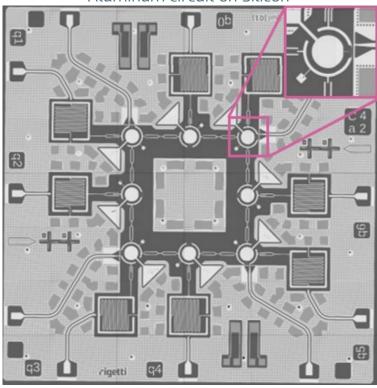






Our Implementation





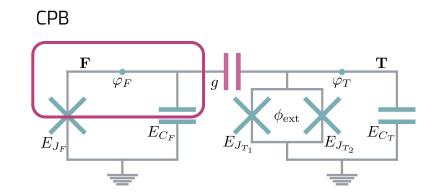
Circuit Quantum Electrodynamics (cQED)

- Superconducting circuits at very low temps (0.01 Kelvin)
- Qubit = Circuit element made with Josephson Junctions (JJ's)



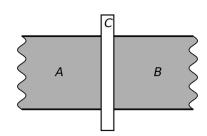
cQED - the transmon qubit(s)

- Josephson Junction Cooper Pair Box
- Transmon regime: $E_J \gg E_C$
- JJ's are nonlinear.
- Nonlinearity→two-level subspace→qubit
- Qubits coupled to linear resonators for state readout
- Jaynes-Cummings Hamiltonian:
 Atomic transition coupled to cavity mode
- Two types of transmons: Fixed & Tunable





Josephson Effect



$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \begin{pmatrix} qV/2 & K \\ K & -qV/2 \end{pmatrix} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$$

Fundamental commutation relation of cQED

$$[\hat{\Phi},\hat{Q}] = i\hbar$$

Equations of Motion

$$I_C = \dot{\rho}_A = -\dot{\rho}_B = I_0 \sin(\phi)$$

$$\dot{\phi} = \dot{\theta}_A - \dot{\theta}_B = \frac{qV}{\hbar}$$



Fixed SC Qubit

Equation of motion

$$I_C = C \frac{dV_C}{dt} = \frac{\hbar}{q} \ddot{\phi}$$

80

$$I_J = I_0 \sin(\phi)$$

$$\frac{\hbar}{q}\ddot{\phi} + I_0 \sin(\phi) = I_b$$

 Using Lagrangian formalism and fundamental commutation relation to arrive at Hamiltonian

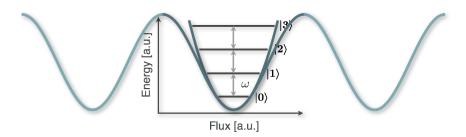
$$H = E_C \hat{n}^2 - E_J \cos(\hat{\phi}) - E_J \frac{I_b}{I_0} \hat{\phi}$$



Washboard and SC-Qubits

Taylor expansion: Harmonic Oscillator

$$H = E_C \hat{n}^2 + \frac{1}{2} E_J \cos(\phi_0) (\hat{\phi} - \phi_0)^2$$



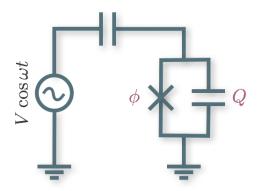
Washboard and SC-Qubits

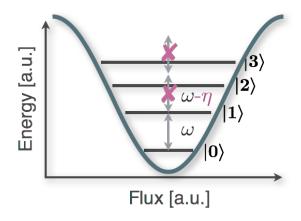
Taylor expansion: Harmonic Oscillator

$$H = E_C \hat{n}^2 + \frac{1}{2} E_J \cos(\phi_0) (\hat{\phi} - \phi_0)^2$$

- Higher order correction: Anharmonic spectrum
- Ground- and 1st excited state form Qubit states
- Transmon Regime

$$E_J \gg E_C$$







Tunable SC Qubit

- Phase sensitive SQUID
- Hamiltonian

$$\phi_{\mathrm{ext}}$$
 ϕ_{ext}
 $E_{J_{T_2}}$
 E_{C_T}

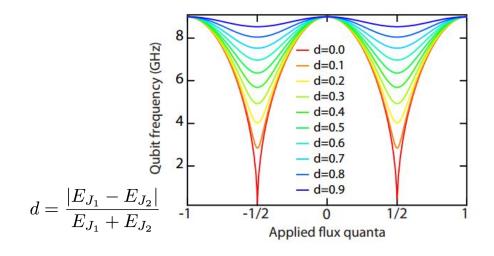
$$H = 4E_C \hat{n}^2 - E_{J_1} \cos(\hat{\phi} - \varphi_{\text{ext}}) - E_{J_2} \cos(\hat{\phi})$$

Symmetric junctions: $E_{J1} = E_{J2}$

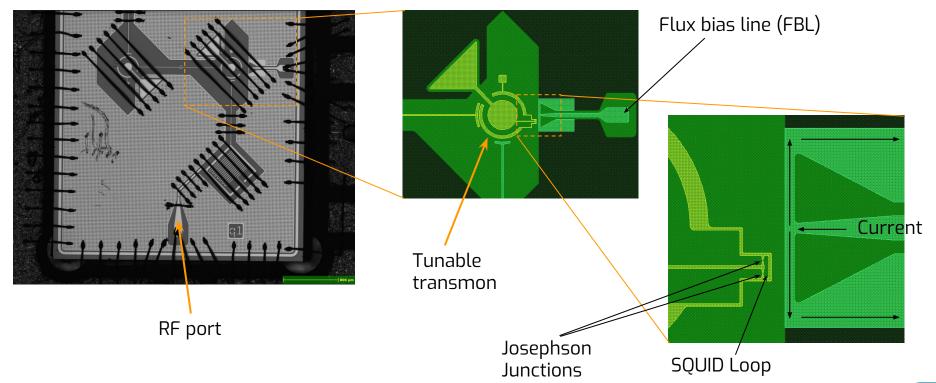
Phases cancel completely at $\Phi = \Phi_0/2$

Asymmetric junctions: $E_{J1} = E_{J2}$

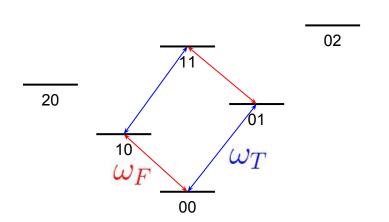
Phases cancel incompletely at $\Phi = \Phi_0/2$



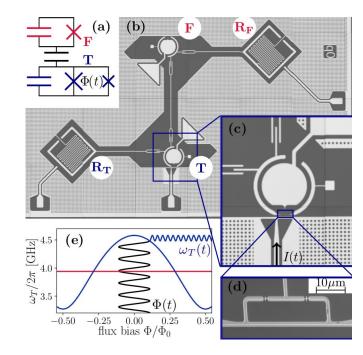
Experimental Device



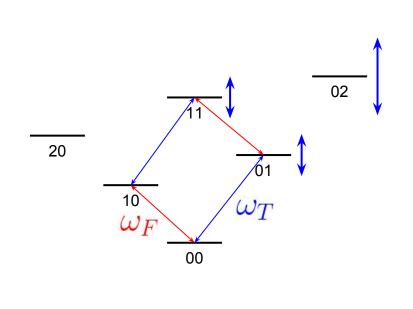




$$\Phi(t) = \overline{\Phi} + \widetilde{\Phi}\cos(\omega_p t + \theta_p)$$

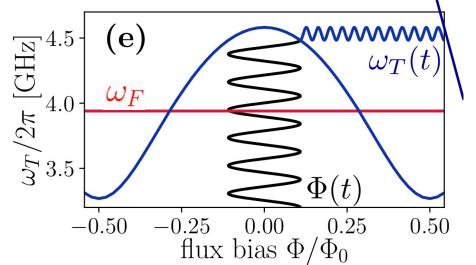




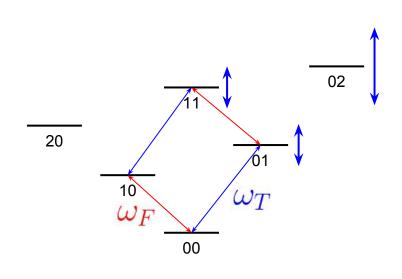


$$\Phi(t) = \overline{\Phi} + \widetilde{\Phi}\cos(\omega_p t + \theta_p)$$

$$\omega_T(t) \approx \overline{\omega_T}(\widetilde{\Phi}) + \widetilde{\omega_T}(\widetilde{\Phi}) \cos(2\omega_p t + 2\theta_p)$$



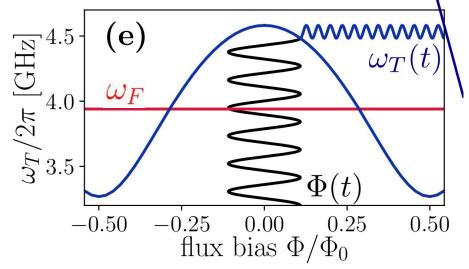




Resonance when $(\omega_T - \overline{\omega_T}) + 2\omega_p = \Delta_{ij}$ for some Δ between levels i, j

$$\Phi(t) = \overline{\Phi} + \widetilde{\Phi}\cos(\omega_p t + \theta_p)$$

$$\omega_T(t) \approx \overline{\omega_T}(\widetilde{\Phi}) + \widetilde{\omega_T}(\widetilde{\Phi}) \cos(2\omega_p t + 2\theta_p)$$





$$|02\rangle$$
 $|11\rangle$
 $|20\rangle$
 $|11\rangle$
 $|20\rangle$
 $|30\rangle$
 $|30\rangle$
 $|30\rangle$
 $|30\rangle$

$$iSWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Resonance when

$$(\omega_T - \overline{\omega_T}) + 2\omega_p = \Delta_{ij}$$

for some Δ between levels i, j



Randomized Benchmarking

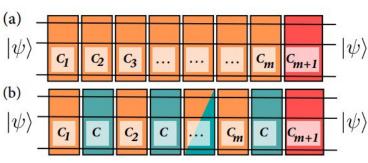
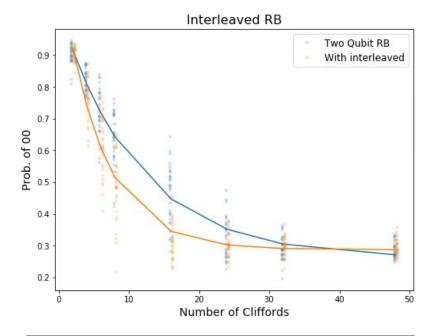


Image from Magesan

Interleaved RB

- A. Run sequences of 20 Cliffords
- B. Run sequences with gate C interleaved

Attempt to isolate the infidelity due to C

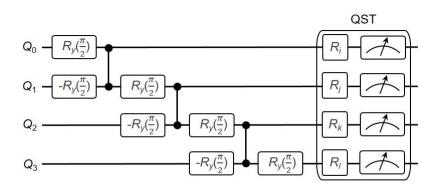


2Q gate type	Avg error per 2Q gate
iSwap	5.9%
CZ ₀₂	8.5%
CZ ₂₀	8.7%

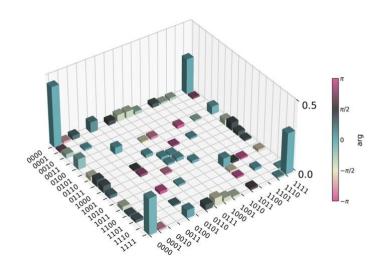


4 qubit entangled state verification

Quantum Circuit

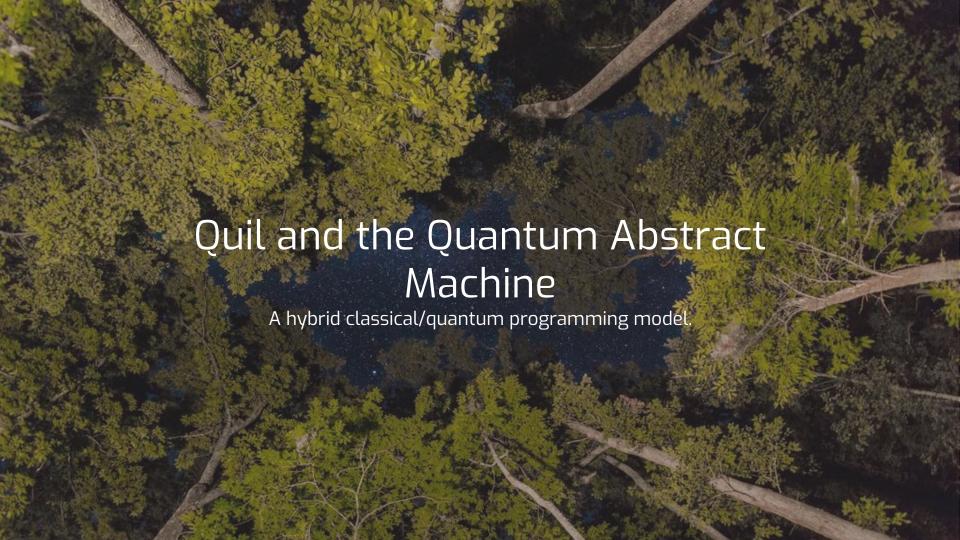


Quantum State Tomography



Fidelity of 79%





FSM & QAM

FSM

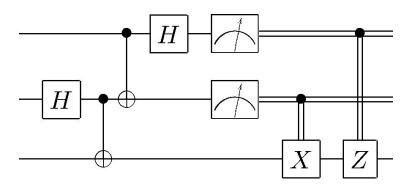
- Classic Bits O and 1 encode the state
- Universal Gate Sets
 - NOT + AND
 - \circ NOT + OR
 - AND + XOR
 - 0 ..
- Execution state (next instruction)

QAM

- Qubits $|0\rangle$ and $|1\rangle$
- Universal Quantum Gate Set
 - CNOT + Single Qubit Gates
- Classic Bits O and 1 encode the state
- Universal Gate Sets
 - NOT + AND
 - \circ NOT + OR
 - AND + XOR
 - 0 ...
- Execution state (next instruction)



Quil is **portable** and **hybrid**.



The Quil Programming Model

Targets a Quantum Abstract Machine (QAM)

- > **Quil** is the instruction language and is how you interact with the machine
- > It is a syntax for representing state transitions.



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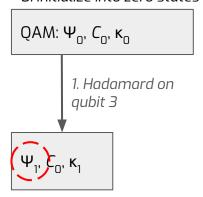
```
\begin{array}{lll} \Psi : \mbox{Quantum state (qubits)} & \rightarrow \mbox{quantum instructions} & \# \mbox{ Quil Example} \\ \mbox{$C$: Classical state (bits)} & \rightarrow \mbox{classical and measurement instructions} & \# \mbox{ Quil Example} \\ \mbox{$H$ 3$} \\ \mbox{$K$: Execution state (program)} & \rightarrow \mbox{control instructions (e.g., jumps)} & \mbox{$MEASURE 3 [4]$} \\ \mbox{$JUMP-WHEN @END [5]} \\ \mbox{$\cdot$} & \mbox{$\cdot$} & \mbox{$\cdot$} \\ \mbox{$\cdot$} & \mbox{$\cdot$} & \mbox{$\cdot$} & \mbox{$\cdot$} \\ \mbox{$\cdot$} & \mbox{$\cdot$} & \mbox{$\cdot$} & \mbox{$\cdot$} & \mbox{$\cdot$} \\ \mbox{$\cdot$} & \mbox{$\cdot$} & \mbox{$\cdot$} & \mbox{$\cdot$} & \mbox{$\cdot$} \\ \mbox{$\cdot$} & \mbox{$\cdot$} & \mbox{$\cdot$} & \mbox{$\cdot$} & \mbox{$\cdot$} & \mbox{$\cdot$} & \mbox{$\cdot$} \\ \mbox{$\cdot$} & \mbox{$\cdot$} &
```

The Quil Programming Model

Targets a **Quantum Abstract Machine (QAM)**

- > **Quil** is the instruction language and is how you interact with the machine
- > It is a syntax for representing state transitions.
- Ψ : Quantum state (qubits) \rightarrow quantum instructions
- C: Classical state (bits) → classical and measurement instructions
- κ: Execution state (program)→ control instructions (e.g., jumps)

O. Initialize into zero states



```
# Quil Example
H 3

MEASURE 3 [4]
JUMP-WHEN @END [5]
```

- •
- •
- .

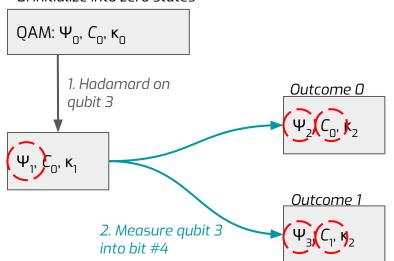


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JUMP-WHEN @END [5]
```

- .
- .
- .

The Quil Programming Model

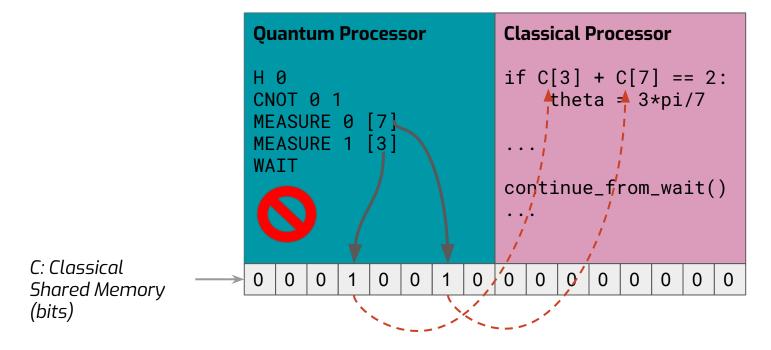
Targets a **Quantum Abstract Machine (QAM)**

- > Quil is the instruction language and is how you interact with the machine
- > It is a syntax for representing state transitions.
- # Quil Example Ψ : Quantum state (qubits) \rightarrow quantum instructions H 3 C: Classical state (bits) → classical and measurement instructions κ: Execution state (program)→ control instructions (e.g., jumps) MEASURE 3 [4] JUMP-WHEN @END [5] O. Initialize into zero states QAM: Ψ_{n} , C_{n} , κ_{n} 1. Hadamard on 3. Jump to end of program Outcome 0 aubit 3 if bit #5 is TRUE Outcome 1 2. Measure qubit 3 into bit #4



Interacting with a Classical Computer

- > The Quantum Abstract Machine has a **shared classical state**.
- > The QAM becomes a practical device with this shared state.
- > Classical computers can take over with classical/quantum synchronization.





Formal Details: The Quil White Paper

arXiv:1608.03355

A Practical Quantum Instruction Set Architecture

Robert S. Smith, Michael J. Curtis, William J. Zeng
Rigetti Computing
775 Heinz Ave.
Berkeley, California 94710
Email: {robert, spike, will}@rigetti.com

Abstract—Quantum computing technology has advanced rapidly in the last few years. Physical systems—superconducting qubits in particular—promise scalable gate-based hardware. Alongside these advances, new algorithms have been discovered that are adapted to the relatively smaller, noisier hardware that will become available in the next few years. These tend to be hybrid classical/quantum algorithms, where the quantum hardware is used in a co-processor model. Here, we introduce an abstract machine architecture for describing these algorithms, along with a language for representing computations on this machine, and discuss a classically simulable implementation architecture. Keywords—quantum computing, software architecture

V	Quil Examples			
	IV-A	Quantum Fourier Transform		
	IV-B	Static and Dynamic Implementation of QVE		
		IV-B1	Static implementation	8
		IV-B2	Dynamic implementation	8
7	A Quantum Programming Toolkit			
	V-A Overview			9



Quantum Teleportation in Quil

If Alice's qubits are 0 and 1

and Bob's is 5
TELEPORT 0 1 5

```
[0]
DEFCIRCUIT TELEPORT A q B:
                               Alice's ancilla q
    # Bell pair
     CNOT
                                       Alice A
     # Teleport
     CNOT
              q [0]
     MEASURE
                                       Bob B
     MEASURE A [1]
     # Classically communicate measurements:
     JUMP-UNLESS @SKIP [1]
     ΧВ
     LABEL @SKIP
     JUMP-UNLESS @END [0]
     Z B
     LABEL @END
```



Teleportation Truth Table

- Bell State: |00⟩+ |11⟩
- Ancilla: $\alpha |0\rangle + \beta |1\rangle$

State A B q>	CNOT q A	Нq	Classic C-X A B	Classic C-Z q B
α 1000>	α 1000>	$ 00\rangle_{Aq}(\alpha 0\rangle + \beta 1\rangle)_{B}$	$[0, 0]_{Aq}; \alpha 0\rangle + \beta 1\rangle$	α 0>+ β 1>
β (001)	β 101>	111> _{Aq} (α 1>- β 0>) _B	[1, 1] _{Aq} ; α Ο⟩- β 1⟩	α 0>+ β 1>
α 110>	α 110>	$ 10\rangle_{Aq}(\alpha 1\rangle + \beta 0\rangle)_{B}$	[1, 0] _{Aq} ; α 0>+ β 1>	α 0>+ β 1>
β 111⟩	β (011)	$ 01\rangle_{Aq} (\alpha 0\rangle - \beta 1\rangle)_{B}$	[0, 1] _{Aq} ; α 0⟩- β 1⟩	α 0>+ β 1>



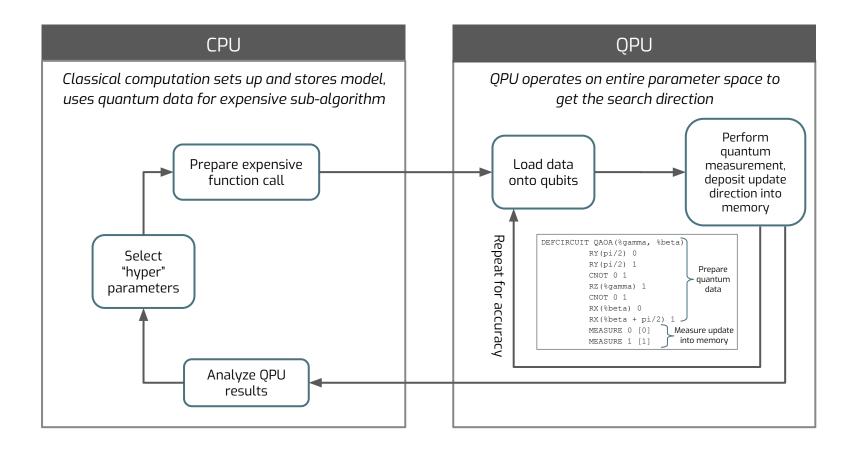
pyQuil generates Quil

```
from pyquil.gates import X, CNOT, H, Z, RX, I
from pyquil.api import QVMConnection
from pyquil.quil import Program
import numpy as np
qvm = QVMConnection()
alice register = 0
ancilla register = 1
flip_correction_branch = Program(X(1))
phase correction branch = Program(Z(1))
prog = (Program()
        .inst(H(0))
        .inst(CNOT(0, 1))
        .inst(RX(0.2 * np.pi, 2))
        .inst(CNOT(2, 0))
        .inst(H(2))
        .measure(0, alice_register)
        .measure(2, ancilla_register)
        .if_then(alice_register, flip_correction_branch)
        .if then(ancilla register, phase correction branch))
qvm.run_and_measure(prog, list(prog.get_qubits()), trials=10)
```

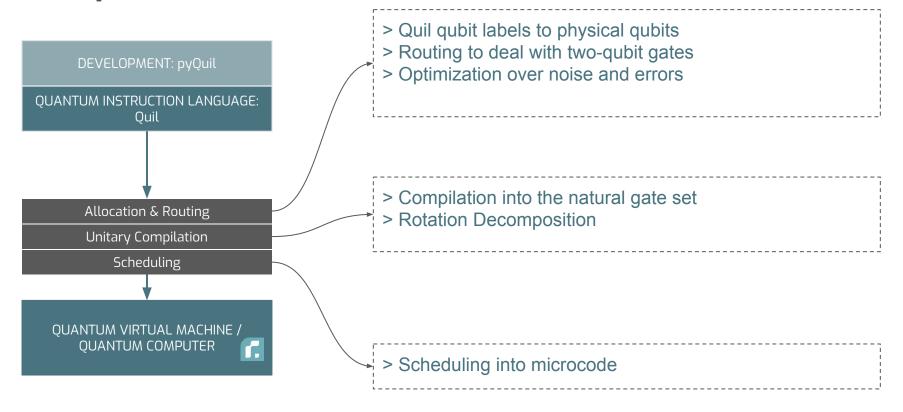
```
H 0
CNOT 0 1
RX(pi/5) 2
CNOT 2 0
H 2
MEASURE 0 [0]
MEASURE 2 [1]
JUMP-WHEN @THEN1 [0]
JUMP @END2
LABEL @THEN1
X 1
LABEL @END2
JUMP-WHEN @THEN5 [1]
JUMP @END6
LABEL @THEN5
Z 1
LABEL @END6
```



Hybrid Quantum Computing



Compilation





Some useful tools

- QVM with up to 37 qubits (on AWS)
- Can simulate arbitrary 1 & 2 qubit noise by definition of Kraus maps

- Compilation layer
 - Specify arbitrary circuit and compile to natural gate set
 - Compression optimizations build in
 - Simple layout optimization: Quantum Circuit to Chip Topology
 - E.g. compiled RB circuit to identity
- Simple prototyping before requesting QPU

```
from pyquil.api import QVMConnection from pyquil.api import QPUConnection

cxn = QVMConnection() cxn = QPUConnection("19Q-Acorn")

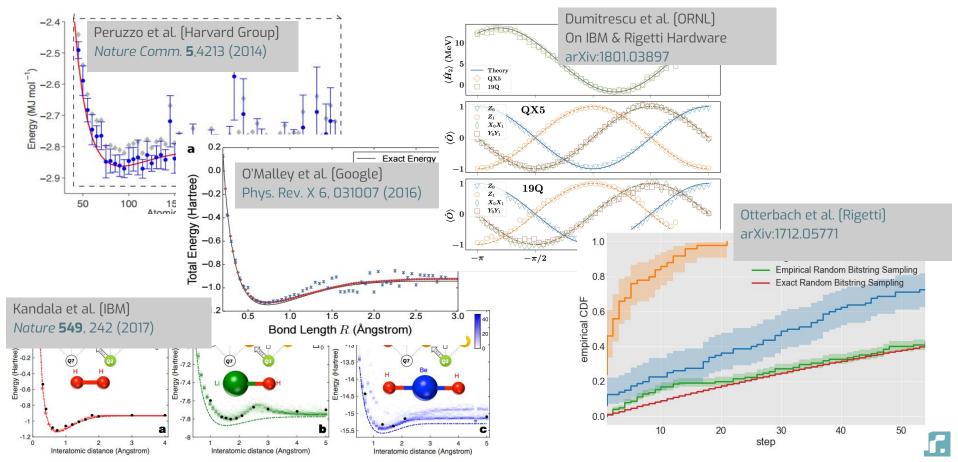
cxn.run_and_measure(prog, ...)
```



ΥΥΘΙΘΈ ΤΟ ΕΝΕΙΘΕΡΙΚΟΙ ΕΝΕΙΘΕΡΙΚΟΙ ΕΝΕΙΘΕΡΙΚΟΙ ΕΝΕΙΘΕΡΙΚΟΙ ΕΝΕΙΘΕΡΙΚΟΙ ΕΝΕΙΘΕΡΙΚΟΙ ΕΝΕΙΘΕΡΙΚΟΙ ΕΝΕΙΘΕΡΙΚΟΙ ΕΝΕΙ 111110100NBQ00100111(L010011Applications1110011

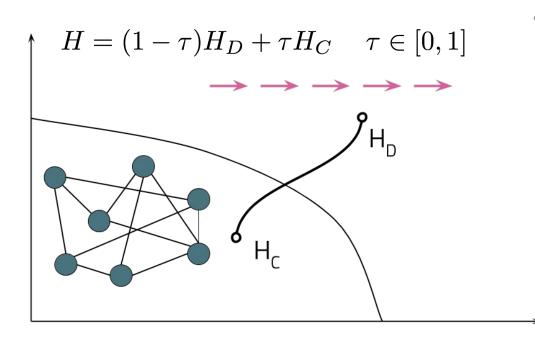
NISQ - Near-term Intermediate Scale Quantum

Preskill, arXiv:<u>1801.00862</u>



QAOA - Quantum Approximate Optimization Algorithm

- Motivated through Adiabatic Quantum Computing
- Encode solution to hard NP-complete problem in ground-state of H_C
- Start at easy to prepare initial state with Hamiltonian H_D



- Typical examples are graph problems
 - Traveling Salesman
 - Portfolio Optimization (Knapsack)
 - N-SAT
 - Sudoku (Exact Cover)

$$H_D = \sum_i \sigma_i^x$$

$$H_C = \sum_{i,j} w_{ij} \sigma_i^z \sigma_j^z + \sum_i h_i \sigma_i^z$$



Trotterization

Gate model of AQC (Farhi, Goldstone, Gutman, arxiv:1411.4028)

$$U = e^{-i((1-\tau)H_D + \tau H_C)t}$$

$$= \lim_{p \to \infty} \left[e^{-i(1-\tau)H_D t/p} e^{-i\tau H_C t/p} \right]^p$$

$$\to \prod_{p=1}^{\infty} e^{-i\beta_p H_D} e^{-i\gamma_p H_C}$$

- Angles β_p , γ_p need not be small
- Theory for $p\rightarrow \infty$ is exact; in practice small p is already good
- No specification on how to find optimal β_p , γ_p

Example: Maxcut

"Maximize disagreement on a colored graph"

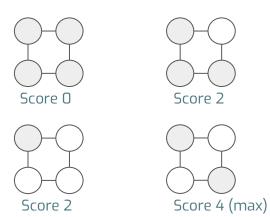




Score 0

Score+1

4-node "ring of disagrees"





Example: Maxcut

- Initial state is ground state of $H_{_{\mathrm{D}}}$: $| \rightarrow \rangle = H^{\otimes n} | 0 \rangle$
- Run the QAOA prescription:

$$|\boldsymbol{\beta}, \boldsymbol{\gamma}\rangle = \prod_{p=1}^{\infty} U_p V_p |H^{\otimes n}|0\rangle$$

Intuitively: Superposition of bitstring configurations

$$|\beta,\gamma\rangle \equiv \sqrt{p_1} + \sqrt{p_2} + ... + \sqrt{p_{16}}$$



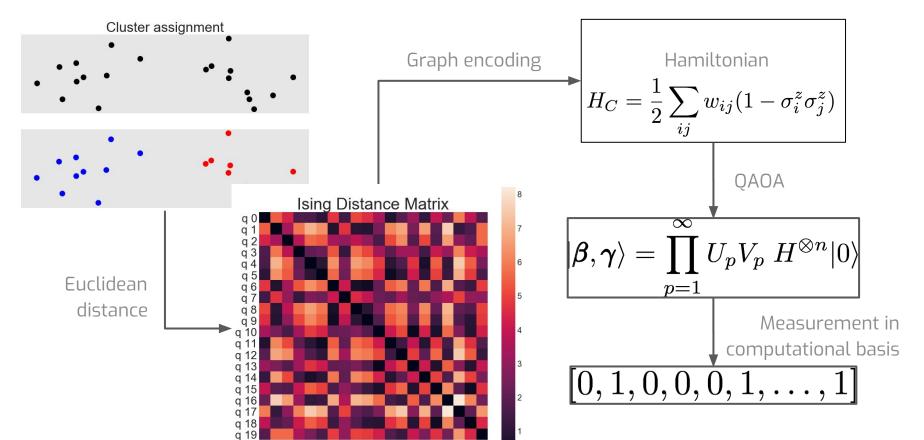
Probability distributions over bit strings

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Cost	Choice $ \beta_1, \gamma_1\rangle$	Choice $ \beta_2, \gamma_2\rangle$	Choice β_{3} , γ_{3} \rangle
0	p = 0.1	p = 0.01	p = 0.2
2	p = 0.3	p = 0.15	p = 0.01
2	p = 0.05	p = 0.2	p = 0.0
4	p = 0.01	p = 0.51	p = 0.25

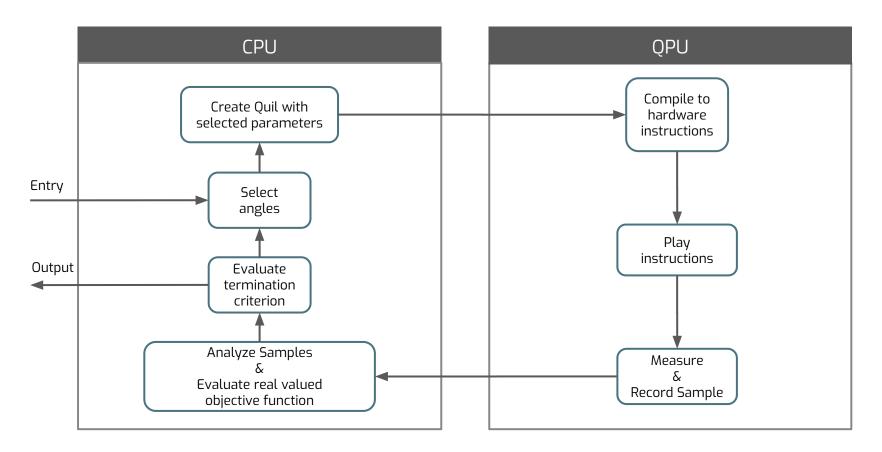


(Weighted) Maxcut as Clustering





When are we done?





Objective Function

Loss/Reward Function:

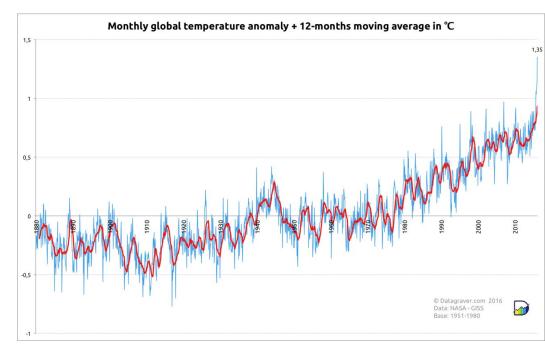
$$c_{\beta,\gamma}: \{0,1\}^n \mapsto \mathbb{R}$$

"quality of a sampled bit-string"

Objective Function

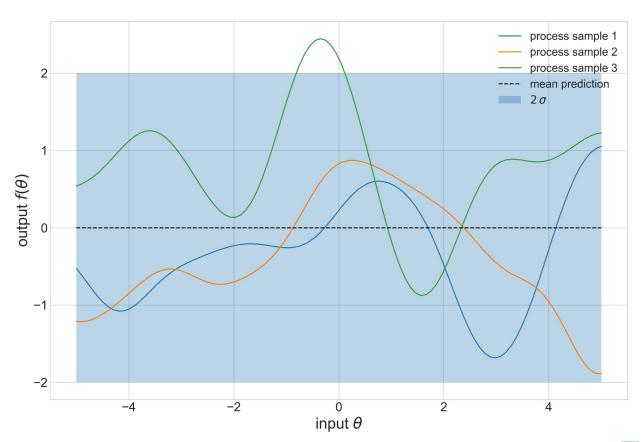
$$f: X_{eta,\gamma} \mapsto \mathbb{R}, \ (eta,\gamma) o \mathtt{STAT}_c(c;eta,\gamma)$$

- Extreme values
- Mean



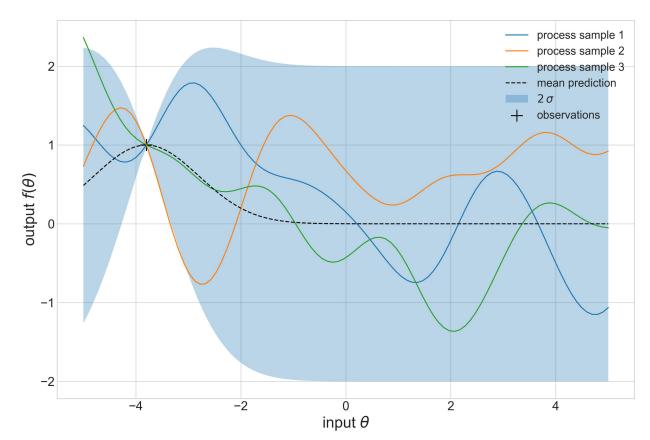
- Find the optimum of objective:
 - No easy access to gradients, need derivative free methods
 - Nelder-Mead
 - Bayesian Methods

 Gaussian Process Prior of objective function



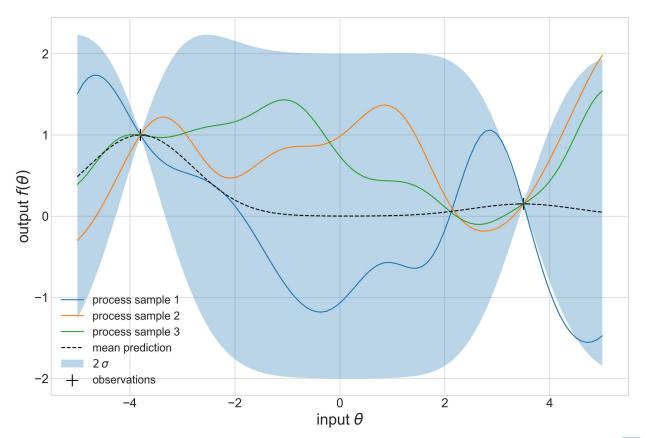


- Gaussian Process Prior of objective function
- Measure and update Prior



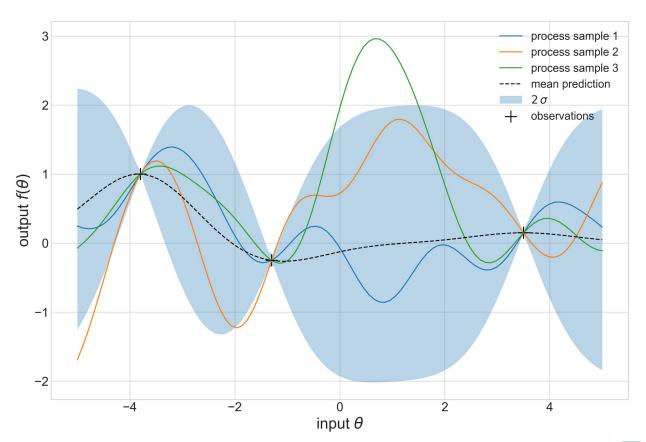


- Gaussian Process Prior of objective function
- Measure and update Prior
- Choose next point to measure and update



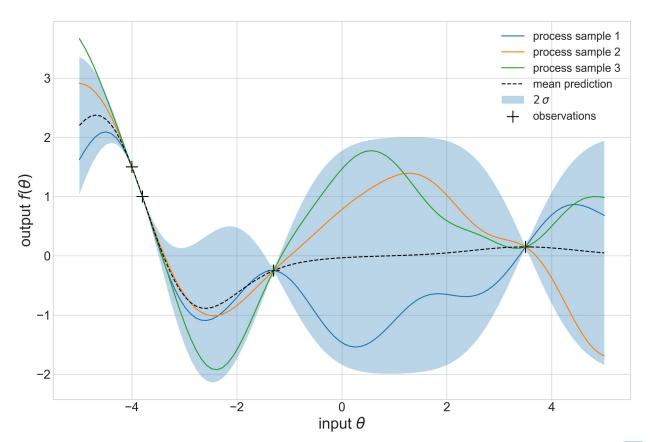


- Gaussian Process Prior of objective function
- Measure and update Prior
- Choose next point to measure and update
- Again



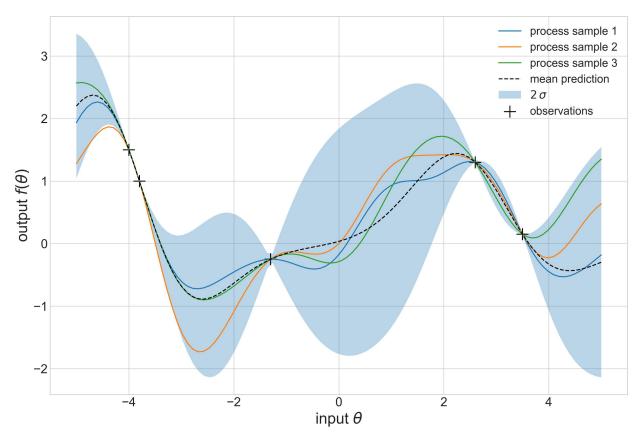


- Gaussian Process Prior of objective function
- Measure and update Prior
- Choose next point to measure and update
- Again
- ...





- Gaussian Process Prior of objective function
- Measure and update Prior
- Choose next point to measure and update
- Again
- ..
- ..



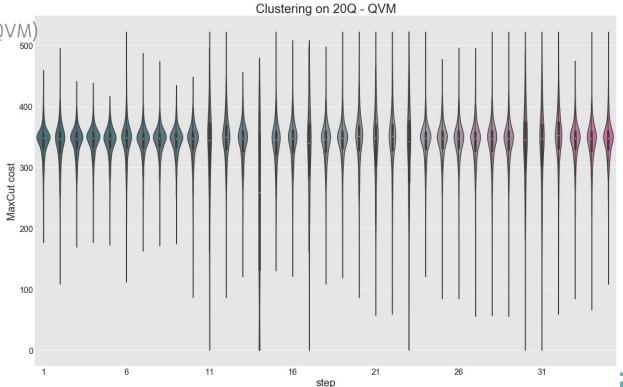


That's how it looks in practice

Fully Connected Graph for clustering

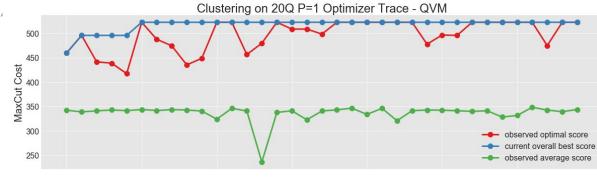
Noiseless Simulator (Rigetti QVM)

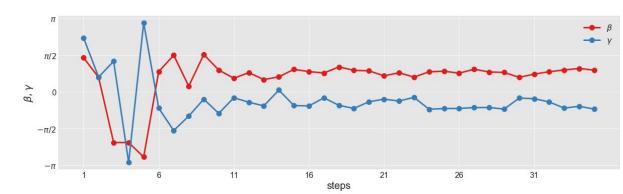
p=1 QAOA



That's how it looks in practice

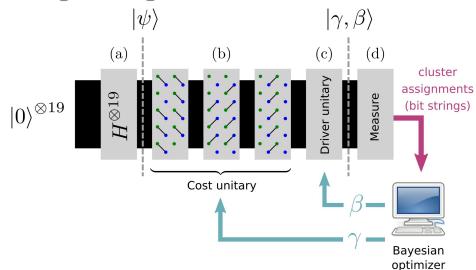
- Fully Connected Graph for clustering
- Noiseless Simulator (Rigetti QVM)
- p=1 QAOA

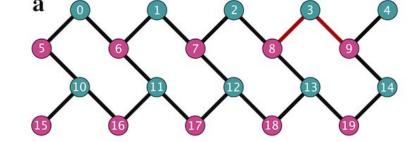


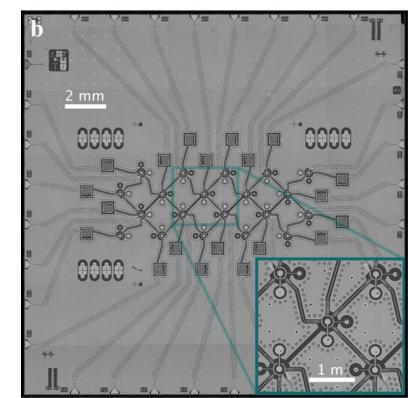


Clustering on a 19-Q Chip

- Chip topology requires smart gate sequence for QAOA to execute all gates on a vertex
- Staggering gate applications according to edge-coloring

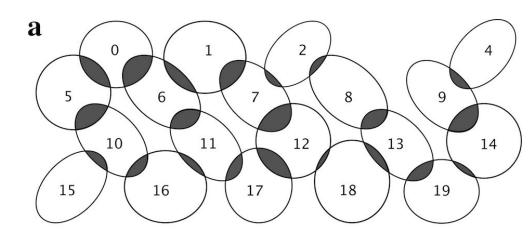


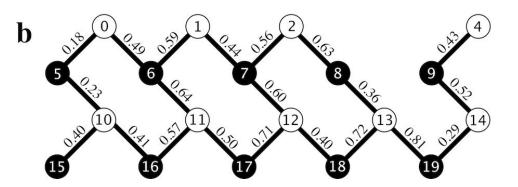




Clustering on a 19-Q Chip

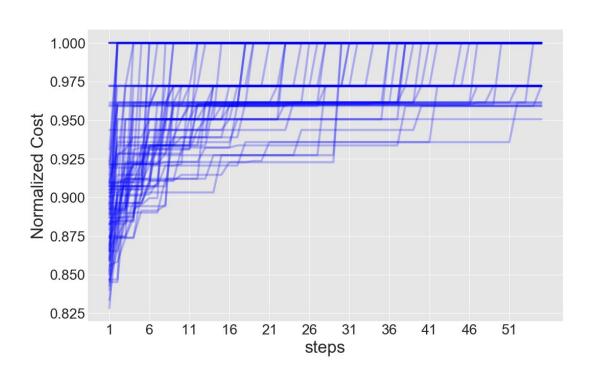
- Moderate coherence times
- Moderate 20 fidelities
- Moderate readout fidelities
- Demonstration with chip specific problem taylored to the topology
- Overlap problem similar to VLSI design







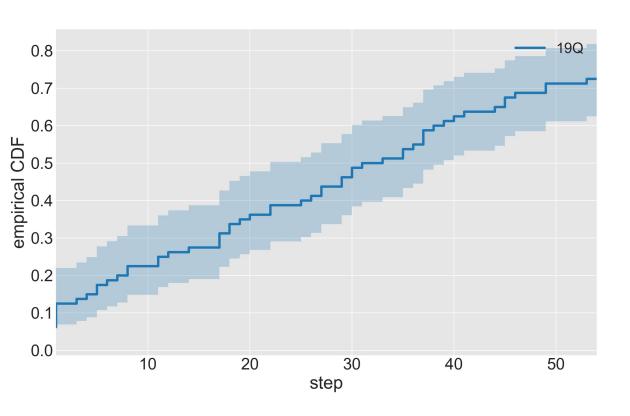
Putting it all together



- 83 trial runs on the QPU
- p=1 QAOA, i.e. single application of U and V
- Algorithm finds the optimum most of the time
- Calculate eCDF form the traces



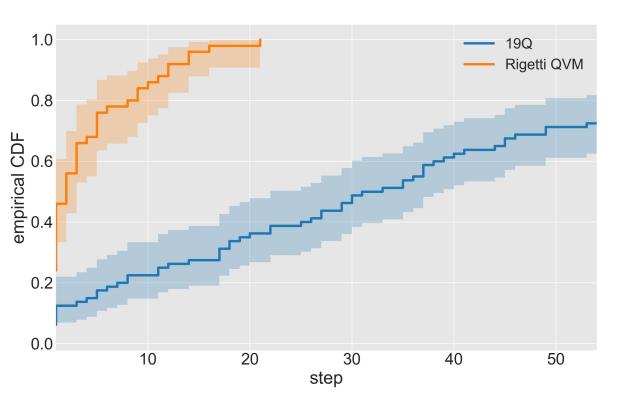
Empirical performance



 Success probability monotonically increases with number of steps.



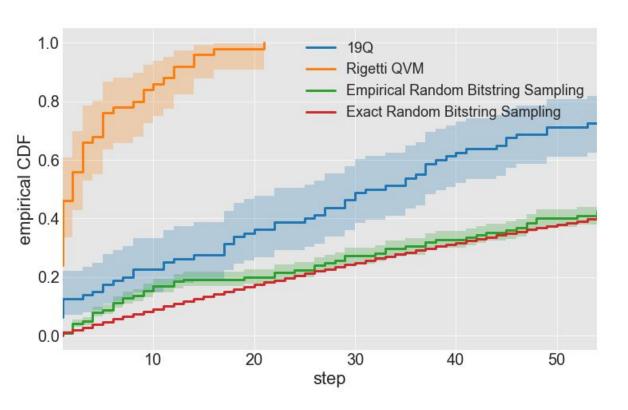
Empirical performance



- Success probability monotonically increases with number of steps.
- Noise in 19Q has a significant impact on performance.



Empirical performance



- Success probability monotonically increases with number of steps.
- Noise in 19Q has a significant impact on performance.
- Approach clearly outperforms random sampling.



Forest



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Sign-Up @ rigetti.com/forest

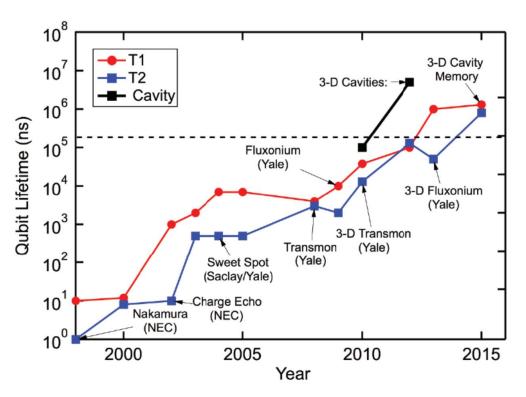
QPU access @ rigetti.com/qpu-request



Spare slides



15 years of exponential performance improvement



Trends in "modern" superconducting qubits:

- > All (or mostly) RF control
- > Dispersive readout
- > 3D cavity resonators



Discrete Fourier Transform (DFT)

Fourier conjugates \mathbf{q},\mathbf{p} (vectors)

Vector: $\mathbf{q} = (q_0, q_1, ..., q_{N-1})$

N is a power of two

Quantum Fourier Transform

Fourier conjugates $|q\rangle,|p\rangle$ (state vectors)

 $|q\rangle = q_0|0\rangle + q_1|1\rangle + ... + q_{N-1}|N-1\rangle$ (Basis vectors explicit)

N is a power of two:

• $N=2^n$ with n qubits

 $DFT[\mathbf{q}] = \mathbf{p}$ $DFT[\mathbf{p}] = \mathbf{q}$

$$QFT|q\rangle = |p\rangle$$

$$QFT|q\rangle = |p\rangle$$

$$p_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} q_j e^{2\pi i j k/N}$$

Quantum Fourier Transform

- Is unitary
- Is faster than DFT
- Is an important subroutine of other algorithms

$$p_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} q_j e^{2\pi i j k/N}$$

Quantum Fourier Transform

- Is unitary ✓
- Is faster than DFT
- Is an important subroutine of other algorithms

$$p_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} q_j e^{2\pi i jk/N}$$

$$\begin{split} |q\rangle &= q_0 |0\rangle + q_1 |1\rangle + ... + q_N |N-1\rangle & \text{Decimal labels} \\ &= q_{0...00} |0...00\rangle + q_{0...01} |0...01\rangle + ... + q_{11...1} |11...1\rangle & \text{Binary labels} \end{split}$$

Quantum Fourier Transform

- Is unitary 🗸
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$$p_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} q_j e^{2\pi i j k/N}$$

$$|q\rangle = q_0|0\rangle + q_1|1\rangle + ... + q_N|N-1\rangle$$
 Decimal labels $= q_{0...00}|0...00\rangle + q_{0...01}|0...01\rangle + ... + q_{11...1}|11...1\rangle$ Binary labels

Fact:
$$\left| \operatorname{QFT} |j_n, ..., j_1 \rangle = \frac{1}{\sqrt{2^n}} \left(|0\rangle + e^{i\varphi_n} |1\rangle \right) \otimes ... \otimes \left(|0\rangle + e^{i\varphi_1} |1\rangle \right) \right|$$

$$\varphi_n \equiv 2\pi (j_1/2^n + j_2/2^{n-1} + \dots + j_n/2)$$

Concrete example: $|q\rangle = |j_3 j_2 j_1\rangle = |101\rangle$

 $\varphi_n \equiv 2\pi (j_1/2^n + j_2/2^{n-1} + \dots + j_n/2)$

Concrete example:
$$|q\rangle = |j_3 j_2 j_1\rangle = |101\rangle$$
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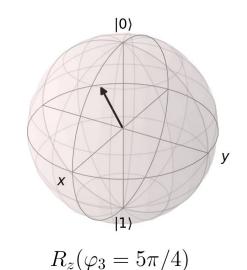
$$\varphi_n \equiv 2\pi (j_1/2^n + j_2/2^{n-1} + \dots + j_n/2)$$

$$QFT|101\rangle = \frac{1}{\sqrt{2^n}} \left(|0\rangle + e^{i2\pi(5/8)}|1\rangle \right) \otimes \left(|0\rangle + e^{i2\pi(1/4)}|1\rangle \right) \otimes \left(|0\rangle + e^{i2\pi(1/2)}|1\rangle \right)$$

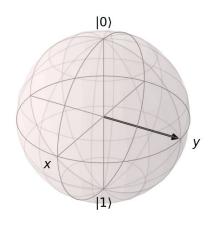
Concrete example:
$$|q\rangle = |j_3 j_2 j_1\rangle = |101\rangle$$

$$\varphi_n \equiv 2\pi (j_1/2^n + j_2/2^{n-1} + \dots + j_n/2)$$

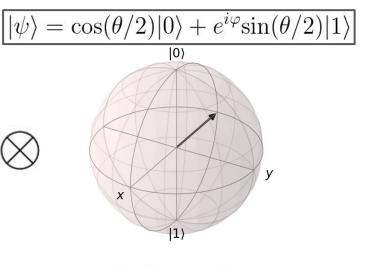
$$QFT|101\rangle = \frac{1}{\sqrt{2^n}} \Big(|0\rangle + e^{i2\pi(5/8)} |1\rangle \Big) \otimes \Big(|0\rangle + e^{i2\pi(1/4)} |1\rangle \Big) \otimes \Big(|0\rangle + e^{i2\pi(1/2)} |1\rangle \Big)$$



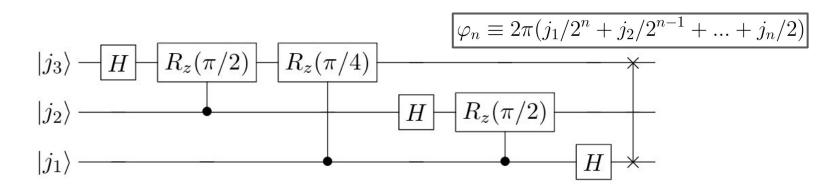


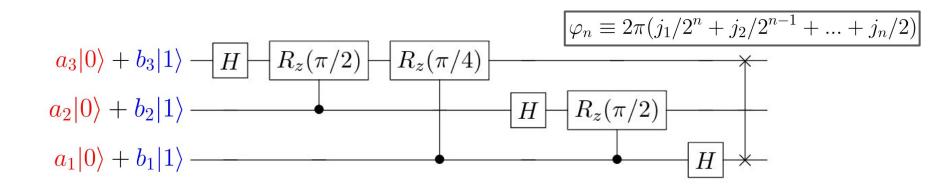


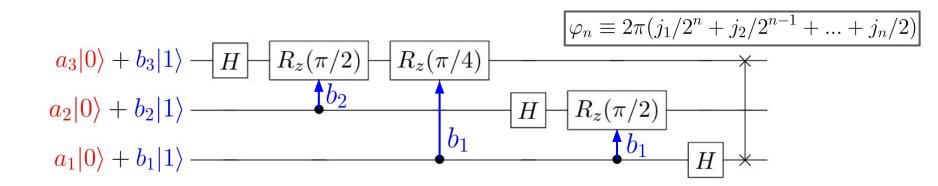
$$R_z(\varphi_2=\pi/2)$$

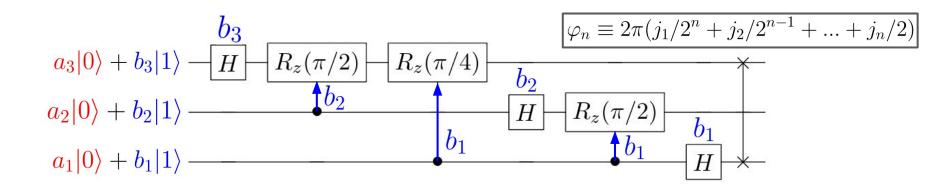


$$R_z(\varphi_1=\pi)$$









$$\begin{array}{c|c} b_3 & \varphi_n \equiv 2\pi(j_1/2^n + j_2/2^{n-1} + \ldots + j_n/2) \\ \hline a_3|0\rangle + b_3|1\rangle & H - R_z(\pi/2) - R_z(\pi/4) \\ \hline a_2|0\rangle + b_2|1\rangle & H - R_z(\pi/2) \\ \hline a_1|0\rangle + b_1|1\rangle & b_1 \\ \hline \end{array}$$

$$p_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} q_j e^{2\pi i j k/N}$$

Fast Fourier Transform: $\Theta(n2^n)$ Quantum Fourier Transform: $\Theta(n^2)$

Quantum Fourier Transform

- Is unitary
- Is faster than Fast Fourier Transform
- Is an important subroutine of other algorithms

Fast Fourier Transform: $\Theta(n2^n)$ Quantum Fourier Transform: $\Theta(n^2)$

Caveats:

- Can't directly read out p_k
 → use as subroutine
- State preparation of $|q\rangle$ is inefficient \rightarrow restricted to simple initial states

$$p_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} q_j e^{2\pi i j k/N}$$

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- Is unitary ✓
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Fast Fourier Transform: $\Theta(n2^n)$ Quantum Fourier Transform: $\Theta(n^2)$

$p_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} q_j e^{2\pi i j k/N}$

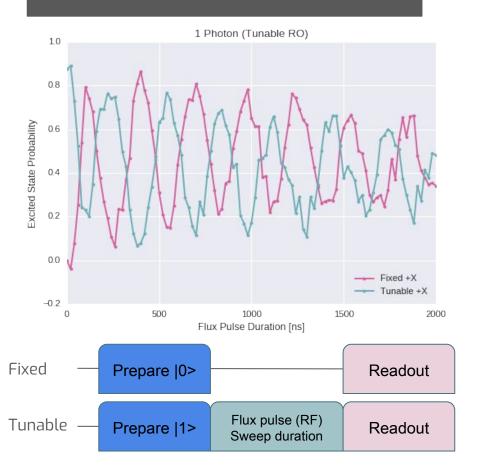
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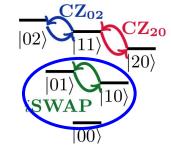
- Can't directly read out p_k
 → use as subroutine
- State preparation of $|q\rangle$ is inefficient
 - → restricted to simple initial states

QFT descendants:

- Phase estimation
- Order finding
- Prime number factorization

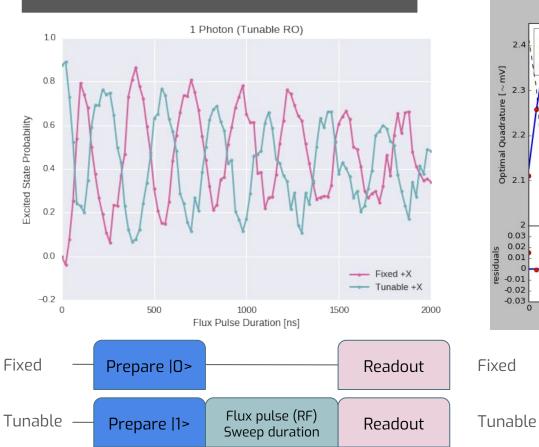
Proof of principle

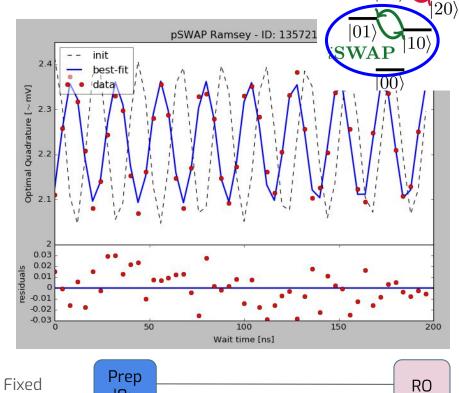






Proof of principle





Vary Wait

Flux

 $\sqrt{i\text{SWAP}}$

RO

|0>

Prep

|1>

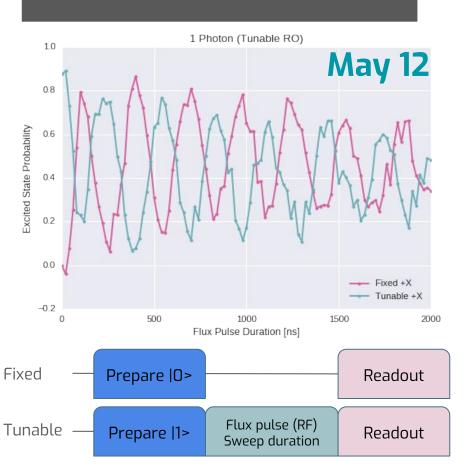
Flux

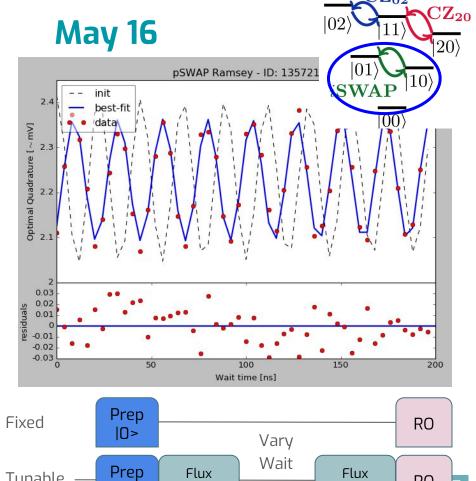
 $\sqrt{i\text{SWAP}}$

 CZ_{02}

 CZ_{20}

Proof of principle





Flux

 $\sqrt{i\text{SWAP}}$

|1>

Tunable

 CZ_{02}

Flux

 $\sqrt{i\text{SWAP}}$

RO

Theory

$$\Phi(t) = \overline{\Phi} + \widetilde{\Phi}\cos(\omega_p t + \theta_p)$$

$$\omega_T(t) \approx \overline{\omega_T}(\widetilde{\Phi}) + \widetilde{\omega_T}(\widetilde{\Phi})\cos(2\omega_p t + 2\theta_p)$$

VAP $|10\rangle$ $|00\rangle$

Resonant frequencies

$$f[\cos\phi_{\text{ext}}(t)] \simeq \bar{f} + \tilde{f}\cos[2(\omega_{p}t + \theta_{p})], \tag{34}$$

$$\bar{f} = f^{(0)}[J_{0}(\tilde{\phi}_{p})] + J_{2}^{2}(\tilde{\phi}_{p})f^{(2)}[J_{0}(\tilde{\phi}_{p})]$$

$$+ J_{2}^{2}(\tilde{\phi}_{p})J_{4}(\tilde{\phi}_{p})f^{(3)}[J_{0}(\tilde{\phi}_{p})] + \frac{1}{4}J_{2}^{4}(\tilde{\phi}_{p})f^{(4)}[J_{0}(\tilde{\phi}_{p})] \tag{35}$$

$$\tilde{f} = -2J_{2}(\tilde{\phi}_{p})\{f^{(1)}[J_{0}(\tilde{\phi}_{p})] + J_{4}(\tilde{\phi}_{p})f^{(2)}[J_{0}(\tilde{\phi}_{p})]$$

$$+ \frac{1}{2}J_{2}^{2}(\tilde{\phi}_{p})f^{(3)}[J_{0}(\tilde{\phi}_{p})] + \frac{2}{3}J_{2}^{2}(\tilde{\phi}_{p})J_{4}(\tilde{\phi}_{p})f^{(4)}[J_{0}(\tilde{\phi}_{p})]\}, \tag{26}$$

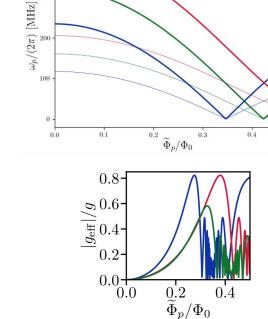
Effective couplings

$$g_{11}^{(n)} = \bar{g}_{11} J_n \left(\frac{\widetilde{\omega}_{T_{01}}}{2\omega_p} \right)$$

$$- \frac{1}{2} \widetilde{g}_{11} \left[J_{n-1} \left(\frac{\widetilde{\omega}_{T_{01}}}{2\omega_p} \right) + J_{n+1} \left(\frac{\widetilde{\omega}_{T_{01}}}{2\omega_p} \right) \right], \quad (41)$$

$$g_{21}^{(n)} = \bar{g}_{21} J_n \left(\frac{\widetilde{\omega}_{T_{01}}}{2\omega_p} \right)$$

$$- \frac{1}{2} \widetilde{g}_{21} \left[J_{n-1} \left(\frac{\widetilde{\omega}_{T_{01}}}{2\omega_p} \right) + J_{n+1} \left(\frac{\widetilde{\omega}_{T_{01}}}{2\omega_p} \right) \right], \quad (42)$$



Didier & Rigetti arXiv:1706.06566

Why do we need to schedule?

- Quil has <u>no</u> notion of time or synchronization.
- But time and synchronization are very important.
- What are our options?

Give up; Admit the physicists are better

"Program" with buttons and wires.

Include *ad hoc* synchronization instructions

Extend Quil to "know" about time.

Compile Quil into some temporal representation

Add machine-specific directives.

Pros:

Maximal control

Cons:

- Difficult to reason about
- Nixes the idea of an abstraction
- Difficult to automate
- Have to think about hardware

Pros:

- Directly addresses the issue
- Still an abstract framework

Cons:

- Extremely complicated!
- Difficult to reason about
- Not easily extensible
- Hard to implement
- Loses the "essence"

Pros:

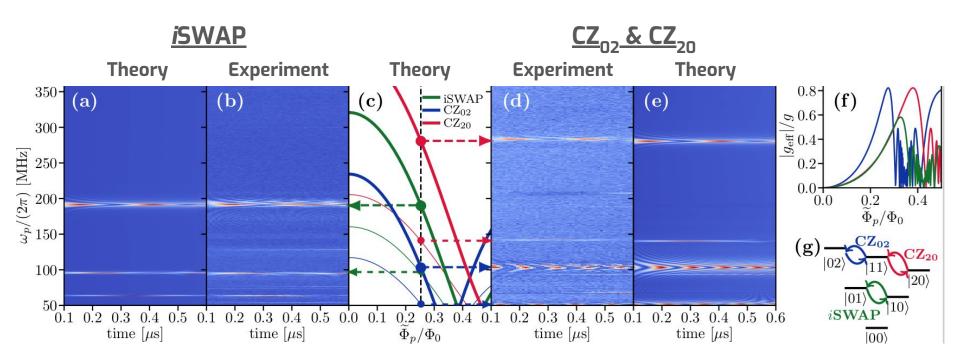
- Remains abstract
- Adds control as necessary
- Extensible!
- Keeps Quil "clean"

Cons:

- Compilation is more difficult
- Performance characterization is machine-specific

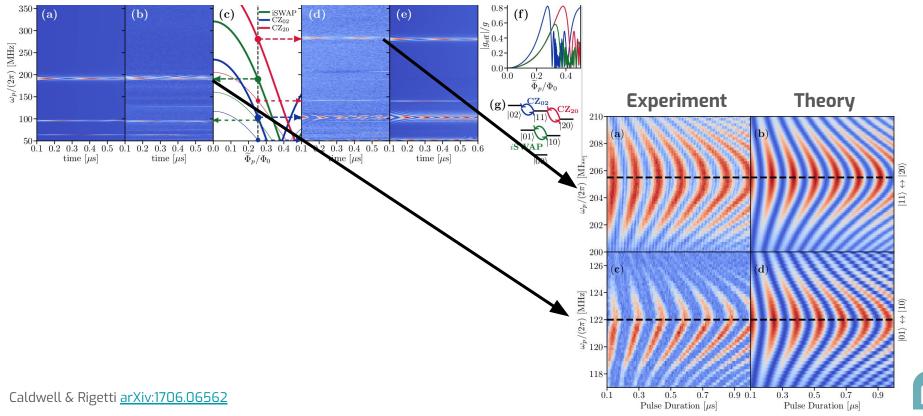


Parametric entangling gates





Parametric entangling gates





Parametric entangling gates

TABLE II. Characteristics of the two-qubit CZ gates performed between neighboring qubit pairs (Q_0, Q_1) , (Q_1, Q_2) , and (Q_2, Q_3) . g represents the qubit-qubit coupling, $\Delta_{11\leftrightarrow02}$ the detuning between $|11\rangle$ and $|02\rangle$, $\Delta_{11\leftrightarrow20}$ the detuning between $|11\rangle$ and $|20\rangle$, ω_m the modulation frequency, $\delta\omega$ the effective detuning of the tunable qubit under modulation, $T_{2,\text{eff}}^*$ the effective coherence time of the tunable qubit under modulation, τ the duration of the CZ gate, and \mathcal{F}_{QPT} the two-qubit gate fidelity measured by quantum process tomography. The symbol † denotes the transitions used for the gate.

Qubit pair	$g/2\pi$	$\Delta_{11\leftrightarrow02}/2\pi$	$\Delta_{11\leftrightarrow 20}/2\pi$	$\omega_m/2\pi$	$\delta\omega/2\pi$	$T_{2,\mathrm{eff}}^*$	τ	$\overline{\mathcal{F}_{QPT}}$
index	(MHz)	(MHz)	(MHz)	(MHz)	(MHz)	(μs)	(ns)	%
$Q_0 - Q_1$	3.8	69.2^{\dagger}	315.0	83.3	281	3.8	278	95
$Q_1 - Q_2$	4.2	187.3^{\dagger}	180.1	82.9	338	3.0	353	93
$Q_2 - Q_3$	4.2	855.1	1240.3^\dagger	199.9	257	5.2	395	91

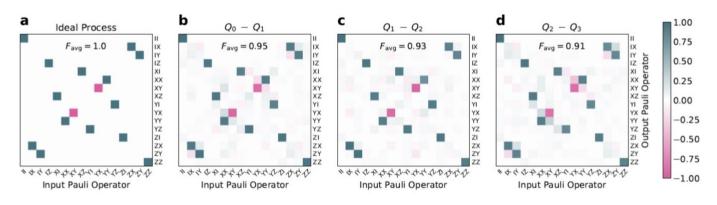


FIG. 3. Quantum process tomography. Process matrices of **a**, the ideal process, and CZ gates between **b**, $Q_0 - Q_1$, **c**, $Q_1 - Q_2$, and **d**, $Q_2 - Q_3$. The achieved average fidelities are measured to be 95%, 93%, and 91%, respectively.



Forest 1.0

June 20

Analytical modeling of parametrically-modulated transmon qubits

Nicolas Didier, Eyob A. Sete, Marcus P. da Silva, and Chad Rigetti Computing, 775 Heinz Avenue, Berkeley, CA 9471 (Dated: June 21, 2017)

Scaling up quantum machines requires developing appropriate models to un their complex quantum dynamics. We focus on superconducting quantum transmons for which full numerical simulations are already challenging at the le thus highly desirable to develop accurate methods of modeling qubit networks it on numerical computations. Using systematic perturbation theory to large ord regime, we derive precise analytic expressions of the transmon parameters. We to the case of parametrically-modulated transmons to study recently-implement activated entangline zates.

I. INTRODUCTION

Scaling up quantum machines is a challenging enterprise that requires accurate modeling of complex quantum dynamics. Precise understanding is crucial to design, manipulate, optimize, and verify the machine. In the field of superconducting quantum computers, transmons [1] 2 are currently widely used as qubits [3] 16 or quantum devices 17-19. Transmons are weakly nonlinear oscillators based on the Cooper pair box, a Josephson junction shunted by a capacitance. The transmon regime corresponds to a large Josephson energy compared to the charging energy- it is a compromise between a large anharmonicity and a weak sensitivity to charge noise. The coherence and gate times of transmons in quantum computing experiments have been steadily improvithe last several years, and transmons are no one leading candidates to an architecture that stringent requirements of fault-tolerapt pantum colputing [20].

Although analytical expression non-interacting transmons are w tood, the accurate description for the bel racting transmons requires the diagon (2a) in or coupled systems (i.e., the charge basis description of the transmons with charge dipole interag erical diagonalization of these systems. omes intractable because a large number of bas accuracy for nontates re necessary to obtain high racting transmons. A more efficient a roa s to use analytical expressions of transtates. Exact diagonalization of the Cooper-pair box h miltonian is achieved with Mathieu functions [21, 22], but manipulating them can be cumbersome. For example, calculating the Fourier transform of Mathieu functions, necessary to describe capacitive couplings, leads to rather complex expressions. An alternative is to consider controlled approximations, such as the approximate diagonalization via standard perturbation theory, which is widely used in quantum mechanics [23]. For transmons, the natural small parameter is the ratio of the charging energy of the Cooper-pair box to the Josephson energy of the junction, as this parameter is typically bellow 2%.

In this paper, we apply
ory to model interacting I
with respect to numerical
analytical expressions are
crosstalk in the dispersivant
analytic expressions to m
transmon qubits to realiz
similar to other proposal
31]. Our theory has bee
predict and simulate iSW
on 2-qubit 151 and 8-qub

We start by presenting single transmon qubit in case of tunable transmon itive coupling of transmon to study flux modulation of Sec. [V] We finally discuss dissipation in Sec. [VI]

II. FIXED-FREQUEN

The circuit of a fixed-fr Josephson junction shunte in Fig. 11 and is governed

$$I_F = 4$$



FIG. 1. Circuit of a fixedable transmon T that are cTransmons are characterized Josephson energy E_J . The pair box corresponds to E_C composed of a SQUID and bias line, $\phi_{\rm ext}(t)$.

Parametrically-Activated Entangling Gates Using Transmon Qubits

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We propose and implement a family of entangling qubit operations activated flux pulses. By parametrically modulating the frequency of a tunable transme selectively actuate resonant exchange of excitations with a statically coupled resonant, neighboring transmon. This direct exchange of excitations between need for mediator qubits or resonator modes, and it allows for the full utilize a scalable architecture. Moreover, we are able to activate three highly-selective sponding to two different classes of entangling gates that enable by using una in SWAP and a controlled-Z rotation. This selectivity is enable by the analysis of the controlled-Z rotation. This selectivity is enable by the analysis of the controlled-Z rotation is a special property of the controlled-Z rotation. This selectivity is enable by the analysis of the controlled-Z rotation. This selectivity is enable by the controlled-Z rotation. The selectivity is enable by the controlled-Z rotation and the controlled-Z rotation. Selectivity is enable by the controlled-Z rotation and the controlled-Z rotation and the controlled-Z rotation. Selectivity is enable by the controlled-Z rotation and the controlled-Z rotat

One of the main challenges in building a scalable superconducting quantum processor architecture is the fly construction of a reliable two-qubit gate. There a two main approaches to achieving this goal using tra mons [1]. The first approach utilizes fixed qubits with static coupling where the two-qu ations are activated by applying transverse mi drives [2–8]. While the fixed-frequency quas have long coherence times, this architectu ble to crosstalk. Moreover, the actual qubit gates requires satisfying strient qubit frequencies and anharmonicit. of these issues, scaling to ma n be challenging. The second approach ties transmons, and two-oubit ga frequency-tunable ctivated by tunir oubits into and out of ach a particular tra tion [9-12]. However comes at the expense additional decoher acc annels, thus significantly lims 18 mostly due to magnetic flux iting coherence or thermore sensitive to frequency or gunwanted crossings with neighborus less during gate operations limits the noise. Such crowdingmectivity of the architecture.

on alter, tive to both of the approaches above relies on a my cally modulating couplings or energy levels at a concy corresponding to the detuning between particular energy levels of interest [14-22]. This enables an entangling gate between a qubit and a single resonator [17, 18], a qubit and many resonator modes [22], two transmon qubits coupled by a tunable mediating qubit [12, 21], or two tunable transmons coupled to a mediating resonator [19, 20].

Building on these earlier results, we implement two en-

ting s. es, iSWAP and the gable transmon an region are activated by as the frequency different ransmon states; [10] and (or [02]) for CZ. We den a two-qubit device, but I wardly to multi-qubit are we drive are highly selectif depends both on the amy as well as its frequency. desirable in multi-qubit sy to solving the problem of Parametrically-activated.

bit gates can be realize in, ce and letting them e speed is then directly set I cating transmon qubits wi rather than a single Joseq quency to be tuned via an the loop. The qubits are with a DC flux pulse to a direction to meet the rese sate for the frequency det 22], akin to frequency co ers [27]. We achieve this the transmon qubit frequ the pulse on the flux bias

To explain the physics of qubit gates, we conside qubit capacitively couple as sketched in Fig. 1(a). 'est levels of the transmor

Demonstration of Universal Parametric Entangling Gates on a Multi-Qubit Lattice

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We show that parametric coupling techniques can be used to generate selective entangling interactions for multi-quibit processors. By inducing coherent population exchange between adjacent qubits under frequency modulation, we implement a universal gateset for a linear array of four superconducting qubits. An average process fieldity of F= 93% se estimated for three two-qubit gates via quantum process tomography. We establish the suitability of these techniques for computation by preparing a four-qubit maximally entangled state and comparing the estimated to delity against the expected performance of the individual entangling gates. In addition, we point agilet-qubit register in all possible distarting permutations and monitor the fieldity of a computation one pair of these qubits. Across all such permutations, an average fix of the pulsay of the control of th

All practical quantum computing architectures must address the challenges of gate implementation at scale. Superconducting quantum processors designed with static circuit parameters can achieve high coherence times [1, 2]. For these schemes, however, entangling gates have come at the expense of always-on qubitqubit couplings [3] and frequency crowding [4]. P cessors based on tunable Josephson qubits, mean can achieve minimal residual coupling and fast qubit operations [5, 6]; yet, these systems come flux noise decoherence [7, 8] and computation basis leakage [9–12]. Moreover, the difficult both fixed-frequency and tunable qub pounded as the system size grows. Para tures [13, 14], however, promise the fundamental challenges of puters. By using module niques akin to analog quantum processors [1] schemes allow for frequency-selective entangling between otherwise static, weakly-interacting qubits

Several proposals for parametric logic gates have been experimentally verified in the last decade. Parametric entangling gates have been demonstrated between two flux qubits via frequency modulation of an ancillary qubit [13, 14]; between two transmon qubits via AC Stark modulation of the computational basis [17] and of the non-computational basis [18] with estimated gate fidelity of $\mathcal{F}=81\%$ [18]; between two fixed-frequency transmon qubits via frequency modulation of a tunable bus resonator with $\mathcal{F}=98\%$ [19]; between high quality factor resonators via frequency modulation of one tunable transmon [20–22] with $\mathcal{F}=[60-80]\%$ [22]; and finally, between a fixed-frequency and tunable transmon

via free cey me. Jation of the same tunable transmon with $J \approx 10^{10}$ Jel. Yet, despite these significant adverse there, is yet to be an experimental assessment of the few ity of parametric architectures with a multi-

Here, we implement universal entangling gates via rametric control on a superconducting processor with eant qubits. We leverage the results of Refs. [23, 24] ric drives can be used to resolve on-chip, multi-qubit the processor, we compare the action of parametric CZ gates to the ideal CZ gate using quantum process tomography (QPT) [25-27], estimating average gate fidelities [28, 29] of $\mathcal{F} = 95\%$, 93%, and 91%. Next, the scalability of parametric entanglement is established by comparing the performance of individual gates to the observed fidelity of a four-qubit maximally entangled state. Further, we directly quantify the effect of the remaining six qubits of the processor on the operation of a single two-qubit CZ gate. To do so, we prepare each of the 64 classical states of the ancilla qubit register and, for each preparation, conduct two-qubit OPT. Tracing out the measurement outcomes of the ancillae results in an average estimated fidelity of $\mathcal{F} = 91.6 \pm 2.6\%$ to the ideal process of CZ. Our error analysis suggests that scaling to larger processors through parametric modulation is readily achievable.

Figure 1a shows an optical image of the processor used in our experiment. The multi-qubit lattice consists of alternating tunable and fixed-frequency transmons, each capacitively coupled to its two nearest neighbors to form a ring topology. This processor is fabricated on a high re-



What is a quantum computer?

Machine that natively executes unitary operations on quantum systems

- Generalizes universal classical computer
- Benefits from inherent size of Hilbert spaces
- Better performance on notable hard problems

Classical state	Quantum state
011	$a_{000} 000\rangle + a_{001} 001\rangle + a_{010} 010\rangle + a_{011} 011\rangle + a_{100} 100\rangle + a_{101} 101\rangle + a_{110} 110\rangle + a_{111} 111\rangle$





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- +1 qubit = 2x compute or memory
 - Addressable problem size
 - Energy efficiency

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What is a quantum computer?

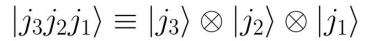
Machine that natively executes unitary operations on quantum systems

- Generalizes universal classical computer
- Benefits from inherent size of Hilbert spaces
- Better performance on notable hard problems
- +1 qubit = 2x compute or memory
 - Addressable problem size
 - Energy efficiency

Interesting properties

- Fully reversible
- No copying an arbitrary state
- Non-deterministic state readout

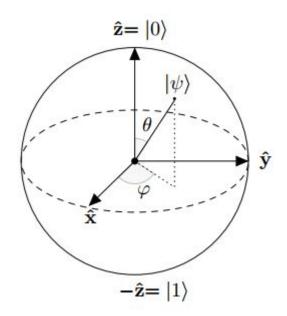
Classical state	Quantum state		
011	$a_{000} 000\rangle + a_{001} 001\rangle + a_{010} 010\rangle + a_{011} 011\rangle + a_{100} 100\rangle + a_{101} 101\rangle + a_{110} 110\rangle + a_{111} 111\rangle$		





One-qubit quantum state

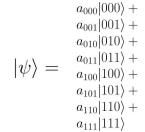
Lives on the surface of the <u>Bloch sphere</u>



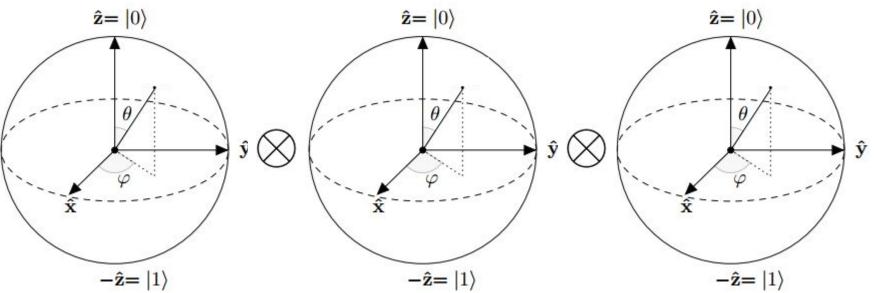
$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\varphi}\sin(\theta/2)|1\rangle$$



Multi-qubit state



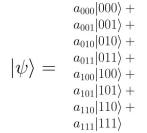




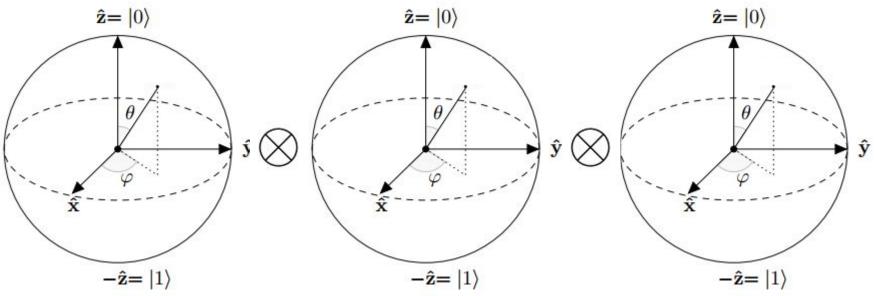
Larger Hilbert space is tensor product of smaller ones



Multi-qubit state



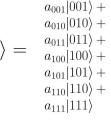
Arbitrary state $|\psi angle$



$$|j_3j_2j_1\rangle \equiv |j_3\rangle \otimes |j_2\rangle \otimes |j_1\rangle$$

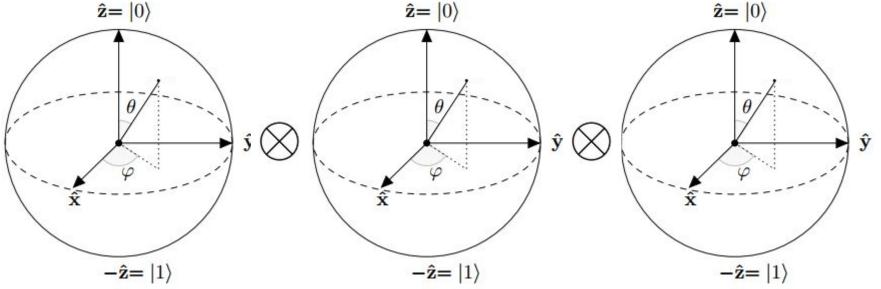


Multi-qubit state



 $a_{000}|000\rangle +$





Basis states:
$$|j_3j_2j_1\rangle \equiv |j_3\rangle \otimes |j_2\rangle \otimes |j_1\rangle$$

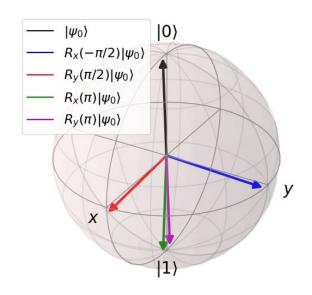
General states:
$$|\psi\rangle \not\equiv |\psi_3\rangle \otimes |\psi_2\rangle \otimes |\psi_1\rangle$$
 (entanglement)

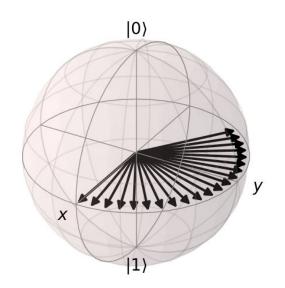


Controlling a quantum state

"Machine that natively executes unitary operations on quantum systems"

Unitaries are rotations





- Qubit frequency f_a
- Qubit precesses in xy (complex) plane at f_a
- X and Y rotations driven with external fields

X and Y rotations by driving

Z rotations by waiting

